A study on Linear Programming Problem Graphical Method

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Abstract

Linear Programming (LP) is one of the most effective mathematical optimization techniques for solving decision-making problems involving limited resources. This paper presents a comprehensive introduction to Linear Programming with particular emphasis on the graphical solution method for problems involving two decision variables. The study explains the fundamental concepts of LP, including objective functions, decision variables, constraints, feasible regions, and optimal solutions. A systematic algorithm for solving LP problems graphically is presented, followed by several illustrative examples covering both maximization and minimization models. The paper also discusses feasible and infeasible solutions, bounded and unbounded regions, basic feasible solutions, and the practical applications of LP in manufacturing, transportation, logistics, engineering, finance, and resource allocation. The graphical method provides an intuitive understanding of optimization by visually identifying feasible regions and optimal corner points. The presented examples demonstrate the effectiveness, accuracy, and educational value of the graphical approach in solving linear optimization problems. This work serves as a useful reference for undergraduate students, educators, and beginners seeking to understand the principles and practical implementation of Linear Programming.

Keywords: **Linear Programming, Graphical Method, Optimization, Feasible Region, Objective Function, Decision Variables, Mathematical Modeling.**

Introduction to Linear Programming

It is an optimization method for a linear objective function and a system of linear inequalities or equations. The linear inequalities or equations are known as constraints. The quantity which needs to be maximized or minimized (optimized) is reflected by the objective function. The fundamental objective of the linear programming model is to look for the values of the variables that optimize (maximize or minimize) the objective function.

We know that in linear programming, we subject linear functions to multiple constraints. These constraints can be written in the form of linear inequality or linear equations. This method plays a fundamental role in finding optimal resource utilization. The word "linear" in linear programming depicts the relationship between different variables. It means that the variables have a linear relationship between them. The word "programming" in linear programming shows that the optimal solution is selected from different alternatives.

We assume the follow the following things while solving the linear programming problems:

- The constraints are expressed in the **quantitative values**
- There is a **linear relationship** between the objective function and the constraints
- The objective function which is also a linear function needs optimization.

Features of Linear Programming

The linear programming problem has the following five features:

Constraints

These are the limitations set on the main objective function. These limitations must be represented in the mathematical form.

Objective function

This function is expressed as a linear function and it describes the quantity that needs optimization.

Linearity

There is a linear relationship between the variables of the function.

Non-negativity

The value of the variable should be zero or non-negative.

Parts of Linear Programming

The primary parts of a linear programming problem are given below:

- Objective function
- Decision variables
- Data
- Constraints

Why we Need Linear Programming?

The applications of linear programming are widespread in many areas. It is especially used in mathematics, telecommunication, logistics, and economics, business and manufacturing fields. The main benefits of using linear programming are given below:

- It provides valuable insights to the business problems as it helps in finding the optimal solution for any situation.
- In engineering, it resolves design and manufacturing issues and facilitates in achieving **optimization of shapes.**
- In manufacturing, it helps to **maximize profits.**
- In the energy sector, it facilitates optimizing the electrical power system.
- In the transportation and logistics industries, it helps in achieving **time and cost efficiency.**

Steps to Solve a Linear Programming Problem

We should follow the steps while solving a linear programming problem graphically.

Step 1 – Identify the decision variables

The first step is to discern the decision variables which control the behaviour of the objective function. Objective function is a function that require optimization.

Step 2 – Write the objective function

The decision variables that you have just selected should be employed to jot down an algebraic expression that shows the quantity we are trying to optimize. In other words, we can say that the objective function is a linear equation that is comprised of decision variables.

Step 3 – Identify Set of Constraints

Constraints are the limitations in the form of equations or inequalities on the decision variables. Remember that all the decision variables are non-negative; i.e., they are either positive or zero.

Step 4 – Choose the method for solving the linear programming problem

Multiple techniques can be used to solve a linear programming problem. These techniques include:

- Simplex method
- Solving the problem using R
- Solving the problem by employing the graphical method
- Solving the problem using an open solver

In this article, we will specifically discuss how to solve linear programming problems using a graphical method.

Step 5 – Construct the graph

After you have selected the graphical method for solving the linear programming problem, you should construct the graph and plot the constraints lines.

Step 6 – Identify the feasible region

This region of the graph satisfies all the constraints in the problem. Selecting any point in the feasible region yields a valid solution for the objective function.

Step 7 – Find the optimum point

Any point in the feasible region that gives the maximum or minimum value of the objective function represents the optimal solution.

Now, that you know what are the steps involved in solving a linear programming problem, we will proceed to solve an example using the steps above.

LPP applications may include production scheduling, inventory policies, investment portfolio, allocation of advertising budget, construction of warehouses, etc. In this article, we would focus on the different components of the output generated by Microsoft excel while solving a basic LPP model. We would solve and discuss four examples together to be aware of the Answer and Sensitivity report and also revisit certain topics covered in the previous article [Elements of a Linear Programming Problem (LPP)].

The main limitations of a linear programming problem (LPP) are listed below:

- It is not simple to determine the objective function mathematically in LPP.
- It is difficult to specify the constraints even after the determination of objectives function.
- There is a possibility that the objective function and constraints may or may not be directly defined by linear in the equality of equations. That means both functions should be linear.
- The main problem while solving a LPP is to determine the suitable coefficient values at each step.
- The assumptions made for LPP are not realistic since they will be taken based on the elements involved in the given situation.
- The solutions obtained in LPP may not be integers all the time.

1. Algorithm & Example-1

Algorithm

Graphical Method Steps (Rule)

Step-1:	Generate the mathematical model of the given LP problem.
Step-2:	a) Replace ≤ and ≥ sign to = in each constraint (inequality to equality) b) Find the two points of each line and draw it on graph. c) For inequality sign of each constraint, decide the area of feasible solution. (For ≤ constraint it is left side of the line and for ≥ constraint it is right side of the line. So for easy understanding, take any point of left side of line and if it satisfies the constraint then mark the left side area otherwise mark the right side area) d) Shade the common portion of the graph.
Step-3:	a) Decide the extreme points (corner points) of the feasible region. b) Calculate the objective function value at each extreme point. c) Find out min or max value (optimal value) of the objective function.

Applications of graphical method:

* Manufacturing Graphical Models has its applications in manufacturing field.
* Finance graphs are visual representations
* Steel production
* Hand wring recognition
* Telecommunication network diagnosis
* Object recognition in images

Advantages of graphical method:

* Graphical methods are quick and easy to use and make visual sense
* Calculations can be done with little or no special software needed.

* Visual test of model (i.e., how well the points line up is an additional benefit

When can you use graphical method of LPP?

The graphical method of solving a linear programming problem can be used when there are only two decision variables. If the problem has three or more variables, the graphical method is not suitable.

Graphical Interpretation

Graphical method or Geometric method, allows solving simple linear programming problems intuitively and visually. This method is limited to two or three problems, decision variables since it is not possible to graphically illustrate more than 3D.

Although in reality only rarely problems arise with two or three decision variables, nonetheless it is very useful this solving methodology. To graphically show possible situation such as the existence of a single optimal solution, alternatives optimal solutions, the nonexistence of solution and unboundedness, it is a visual aid to interpret and understand the simple method algorithm (much more sophisticated and abstract) and concepts surrounding it.

The process phases of problems solving by Graphical method are:

1) Draw a Cartesian coordinate system in which each decision variable is represented by an axis.
2) Establish a measurement scale for each axis suitable to its associated variable.
3) Draw in the coordinate system the constraints of the problem, including non-negativity (which will be the axes themselves). Note that an inequality defines a region which will be the half plane limited by the straight line you have to consider the restriction as an equality, while an equation defines a region that is the straight line itself.
4) The intersection of all regions determines the feasible region or solution space (which is a convex set). If this region is not empty. It will continue with the next step. Otherwise, there is no point that

satisfies all constraints simultaneously, so the problem will not be solved, denominating not feasible.

5) Determine the extreme points (polygon or polyhedron vertices) that shape the feasible region. These points will be the candidates for the optimal solution.
6) Evaluate the objective function at each vertex and the optimal solution will be that (or those) which maximize (or minimize) the resulting value.

Difference between minimization and maximization

A difference between minimization and maximization problems is that:

Maximization problems often have unbounded regions.

Minimization problems often have unbounded regions.

Minimization problems are more difficult to solve than maximization problems.

Important Definitions

Solution The set of values of decision variables $x_j\,(j = 1,2, \dots, n)$ that satisfy the constraints of an LP problem is said to constitute the solution to that LP problem.

Feasible solution The set of values of decision variables $x_j\,(j = 1,2, \dots, n)$ that satisfy all the constraints and non-negativity conditions of an LP problem simultaneously is said to constitute the feasible solution to that LP problem.

Infeasible solution The set of values of decision variables $x_j\,(j = 1,2, \dots, n)$ that do not satisfy all the constraints and non-negativity conditions of an LP problem simultaneously is said to constitute the infeasible solution to that LP problem.

Basic solution For a set of m simultaneous equations in n variables $(n > m)$, a solution obtained by setting $(n - m)$ variables to zero and solving for remaining m equations in m variables is called a basic solution.

The $(n-m)$ variables whose value did not appear in this solution are called non-basic variables and the remaining m variables are called basic variables.

Basic feasible solution A feasible solution to an LP problem which is also the basic solution is called the basic feasible solution. That is, all basic variables assume non-negative values. Basic feasible solutions are of two types:

(a) Degenerate A basic feasible solution is called degenerate if the value of at least one basic variables is zero.

(b) Non-degenerate A basic feasible solution is called non-degenerate if values of all m basic variables are non-zero and positive.

Optimum basic feasible solution A basic feasible solution that optimizes (maximizes or minimizes) the objective function value of the given LP problem is called an optimum basic feasible solution.

Unbounded solution A solution that can indefinitely increase or decrease the value of the objective function of the LP problem is called an unbounded solution.

1. Maximize $\boldsymbol{Z = 12x_1 + 16x_2}$

Subject to

$\boldsymbol{10x_1 + 20x_2 \leq 180}$

$\boldsymbol{8x_1 + 8x_2 \leq 80}$

$\boldsymbol{x_1}$ **and** $\boldsymbol{x_2 \geq 0}$

Sol: Let

$10x_1 + 20x_2 = 180$ .... (1)

x_1	0	12
x_2	6	0

When $x_1 = 0, x_2 = ?$

$10(0) + 20\,(x_2) = 120$

$\Rightarrow x_2 = 6$

When $x_2 = 0, x_1 = ?$

8

$10x_1 + 20(0) = 120$

$\Rightarrow x_1 = 12$

Now $8x_1 + 8x_2 = 80$

x_1	0	10
x_2	10	0

When $x_1 = 0, x_2 = ?$

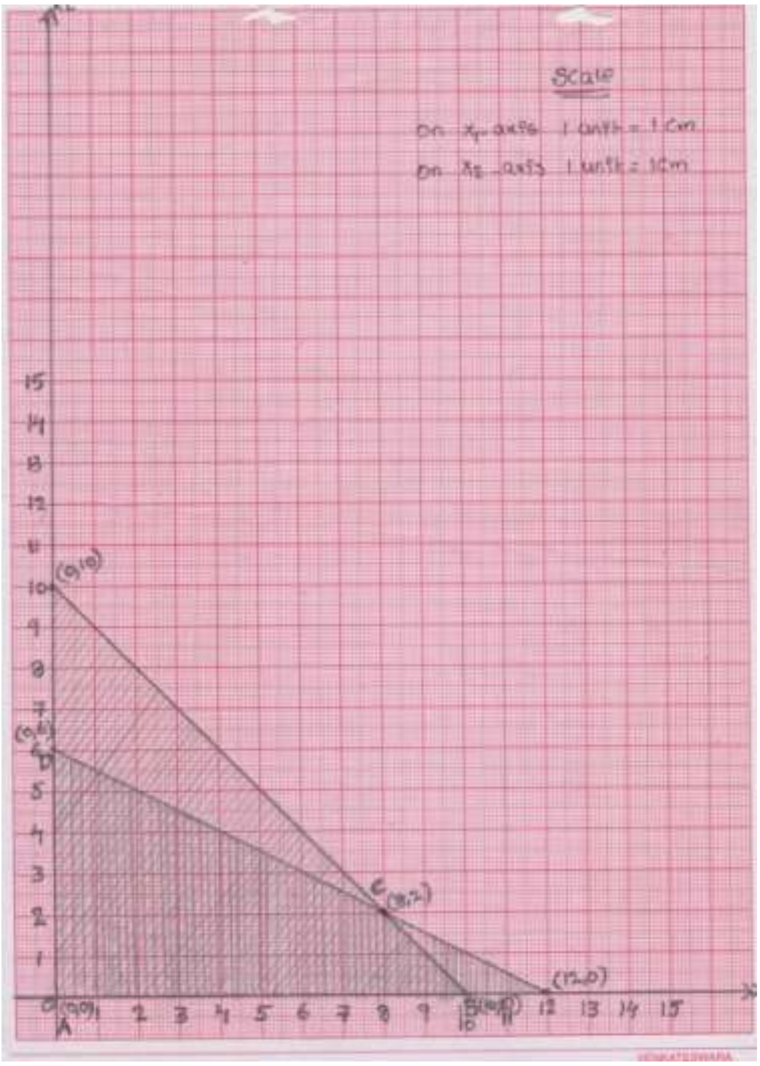
$8(0) + 8x_2 = 80$

$\Rightarrow x_2 = 10$

When $x_2 = 0, x_1 = ?$

$8x_1 + 8(0) = 80$

$\Rightarrow x_1 = 10$



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From Graph

The Festable Region is OBCD

Given $Z = 12x_1 + 16x_2$

At $O(0,0) = 12(0) + 16(0) = 0$

$B(10,0) = 12(10) + 16(0) = 120$

$C(8,2) = 12(8) + 16(2) = 128$

$D(0,6) = 12(0) + 16(6) = 96$

Therefore maximum value of Z occurs at $C(8,2)$

Therefore the solution is

$x_1 = 8, x_2 = 2$

$Z_{max} = 128$

2. Maximize $\boldsymbol{Z = 2x + 5y}$

Subject to

$\boldsymbol{x + 2y \leq 6}$

$\boldsymbol{5x + 3y \leq 45}$

When $\boldsymbol{x, y \geq 0}$

Sol: Given

$x + 2y \leq 16$ .... (1)

$x + 2y = 16$

When $x = 0, y = ?$

$0 + 2y = 16$

$y = 8$

When $y = 0, x = ?$

$x + 2(0) = 16 \Rightarrow$

x	0	16
y	8	0

$x = 16$

10

$5x + 3y = 45$ ..... (2)

When $x = 0, y = ?$

$5(0) + 3y = 45$

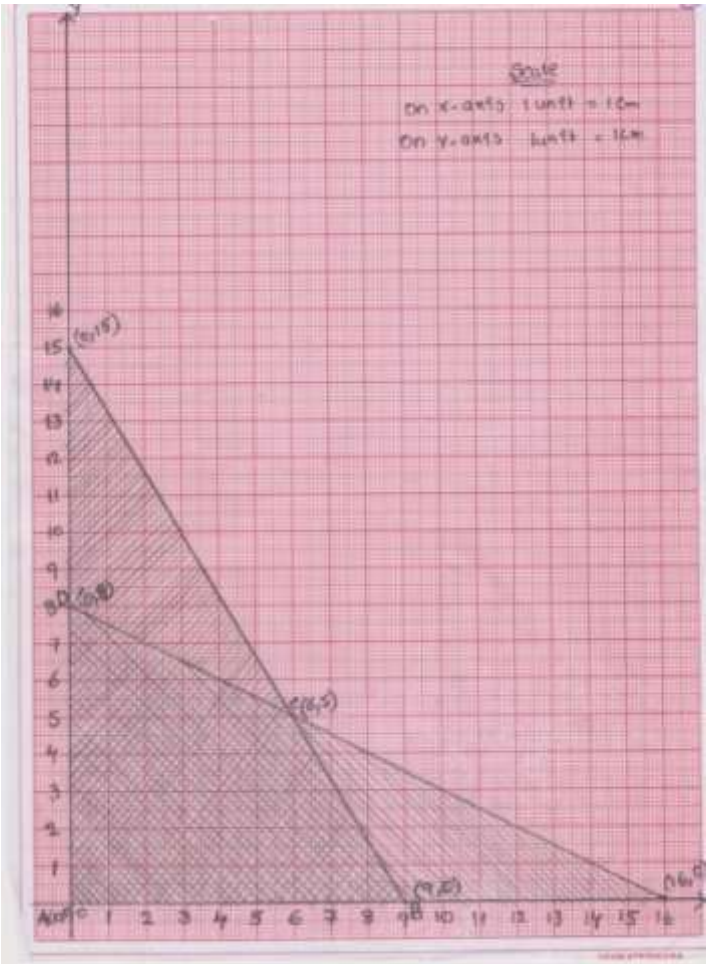
$y = 15$

When $y = 0, x = ?$

$5x + 3(0) = 45$

x	0	9
y	15	0

$x = 9$



From Graph:

At $A(0,0) = 2x + 5y = 2(0) + 5(0) = 0$

$B(9,0) = 2(9) + 5(0) = 18$

$C(6,5) = 2(6) + 5(5) = 37$

$$D(0,8) = 2(0) + 5(8) = 40$$

Therefore maximum value of Z occurs at $D(0,8)$

Therefore the solution is $x_1 = 0, x_2 = 8$ $\quad Z_{max} = 40$

3. Maximize $Z = 3x_1 + 4x_2$ subject to $x_1 - x_2 \leq -1, -x_1 + x_2 \leq 0$ and $x_1, x_2 \geq 0$

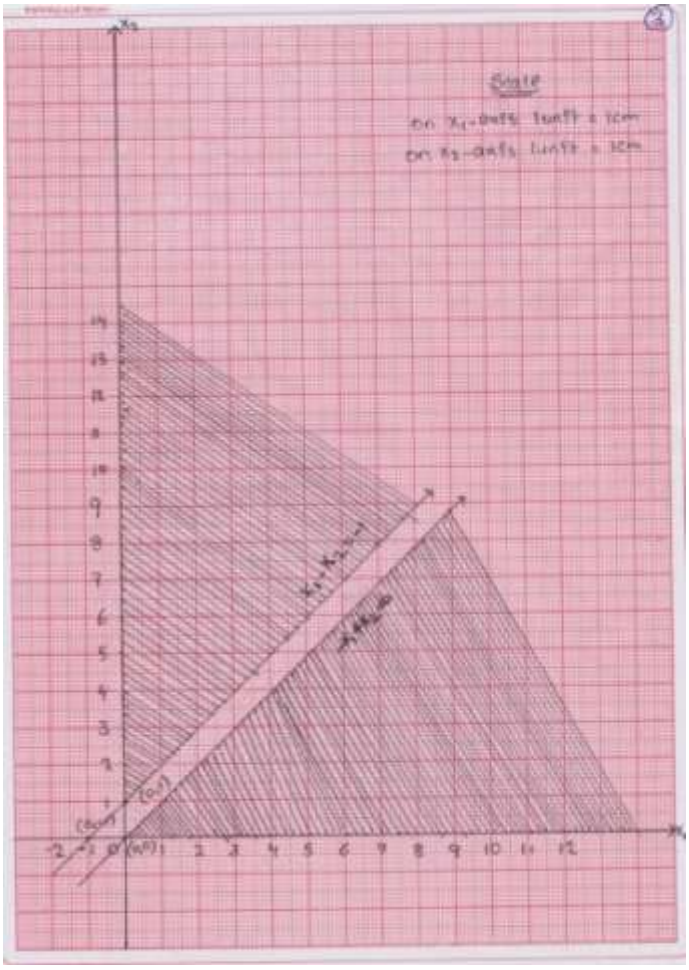
Sol: Since both the decision variable x_1, x_2 are non-negative, the solution lies in the first quadrant of the plane.

Consider the equations $x_1 - x_2 = -1$ and $-x_1 + x_2 = 0$

$x_1 - x_2 = -1$ is a line passing through the points $(0,1)$ and $(-1,0)$

$-x_1 + x_2 = 0$ is a line passing through the point $(0,0)$

Now we draw the graph satisfying the conditions $x_1 - x_2 \leq -1, -x_1 + x_2 \leq 0$ and $x_1, x_2 \geq 0$



There is no common region (feasible region) satisfying all the given conditions.

Hence the given L.P.P. has no solution.

4. Maximize $Z = 2x_1 + 3x_2$ subject to constraints $x_1 + x_2 \leq 30$, $x_2 \leq 12$; $x_1 \leq 20$ and $x_1, x_2 \geq 0$

Sol: Given constraints is

$x_1 + x_2 \leq 30$

$x_2 \leq 12$

$x_1 \leq 20$

Now

$x_1 + x_2 = 30$ .... (1)

$x_2 = 12$ .... (2)

$x_1 = 20$ .... (3)

From (1)

$x_1 + x_2 = 30$

When $x_1 = 0, x_2 = ?$

x_1	0	30
x_2	30	0

$0 + x_2 = 30 \Rightarrow x_2 = 30$

When $x_2 = 0, x_1 = ?$

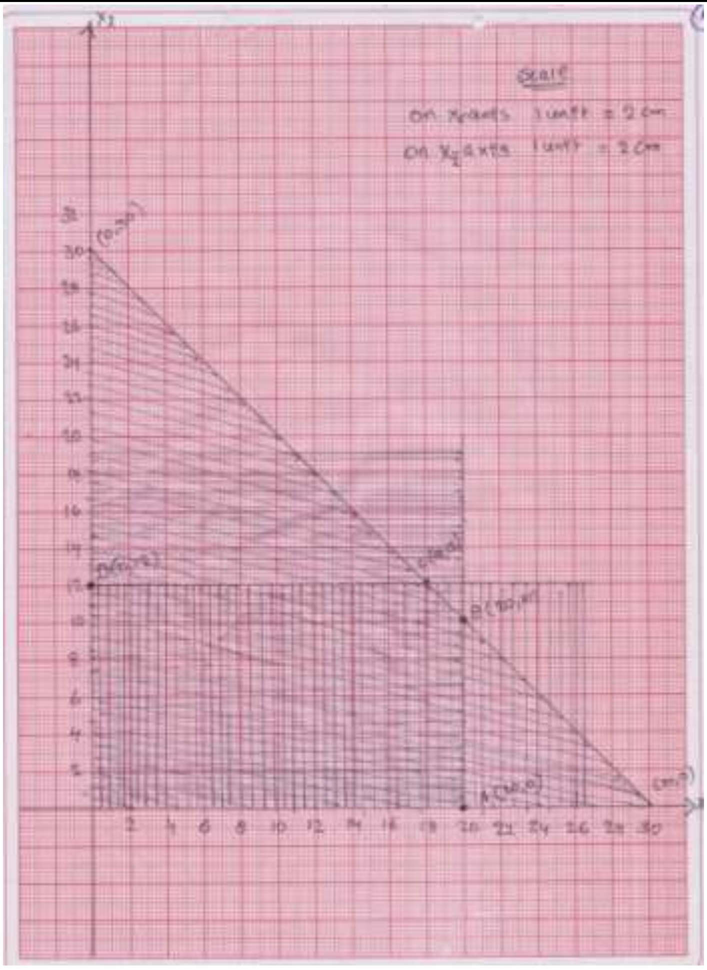
$x_1 + 0 = 30 \Rightarrow x_1 = 30$

From (2)

$x_2 = 12$ is a line parallel to x_1-axis

From (3)

$x_1 = 20$ is a line parallel to x_2- axis



From Graph

At $O(0,0), Z = 2(0) + 3(0) = 0$

$A(20,0), Z = 2(20) + 3(0) = 40$

$B(20,10), Z = 2(20) + 3(10) = 70$

$C(18,12), Z = 2(18) + 3(12) = 72$

$D(0,12), Z = 2(0) + 3(12) = 36$

Therefore maximum value of Z occurs at $C(18,12)$

Therefore the solution is

$x_1 = 18, x_2 = 12$

$Z_{max} = 72$

5. $x + y \leq 40$ Subject to $2x + y \leq 60$ $x \geq 0, y \geq 0$

Sol: Given $x + y = 40$ ..... (1)

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x	0	40
y	40	0

Where $x = 0, 0 + y = 40$

$x + 0 = 40$

When $y = 0, x = 40$

$2x + y = 60$ .... (2)

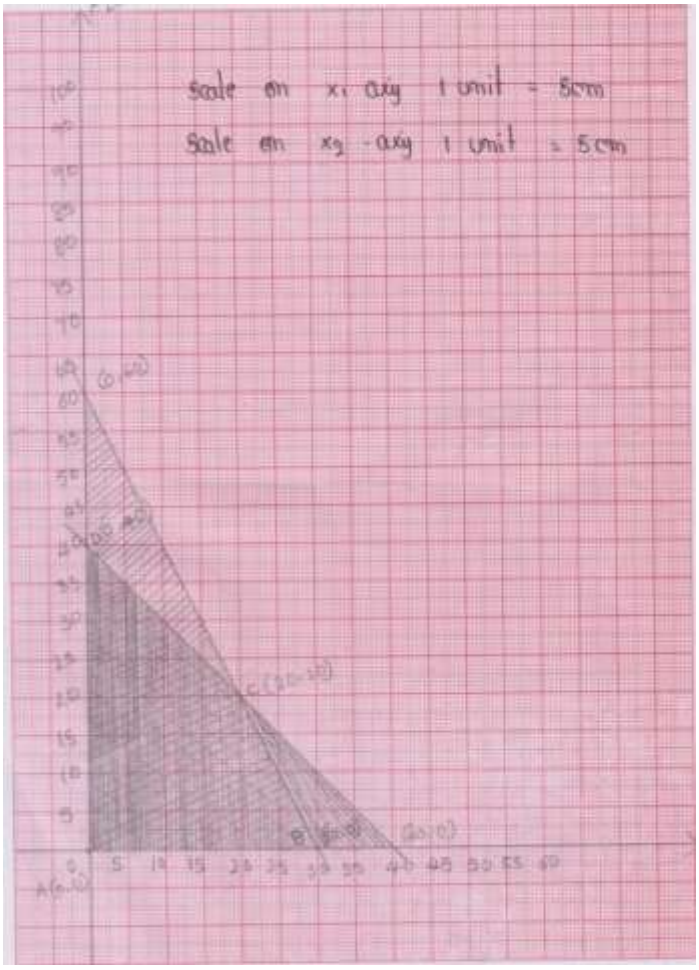
When $x = 0, 2(0) + \ y = 60$

x	0	30
y	60	0

$2x + 0 = 60$

When $y = 0, x = 30$

max $\ Z = 8x + y$



From Graph

$Z(A) = 8(0) + 0 = 8$

$Z(B) = 8(30) + 0 = 240$

$Z(C) = 8(20) + 20 = 160 + 20 = 180$

$Z(D) = 8(0) + 40 = 0 + 40 = 40$

Therefore maximum value of Z occurs at $B = (30,0)$

$Z\ maximum = 240$

6. $Z = x + y$ Subject to $2x + y \leq 50$ $x + 2y \leq 40$

Sol: Given $2x + y = 50$ .... (1)

x	0	25
y	50	0

$x = 0; 2(0) + y = 50 \Rightarrow y = 50$

$y = 0; 2x + 0 = 50 \Rightarrow x = 25$

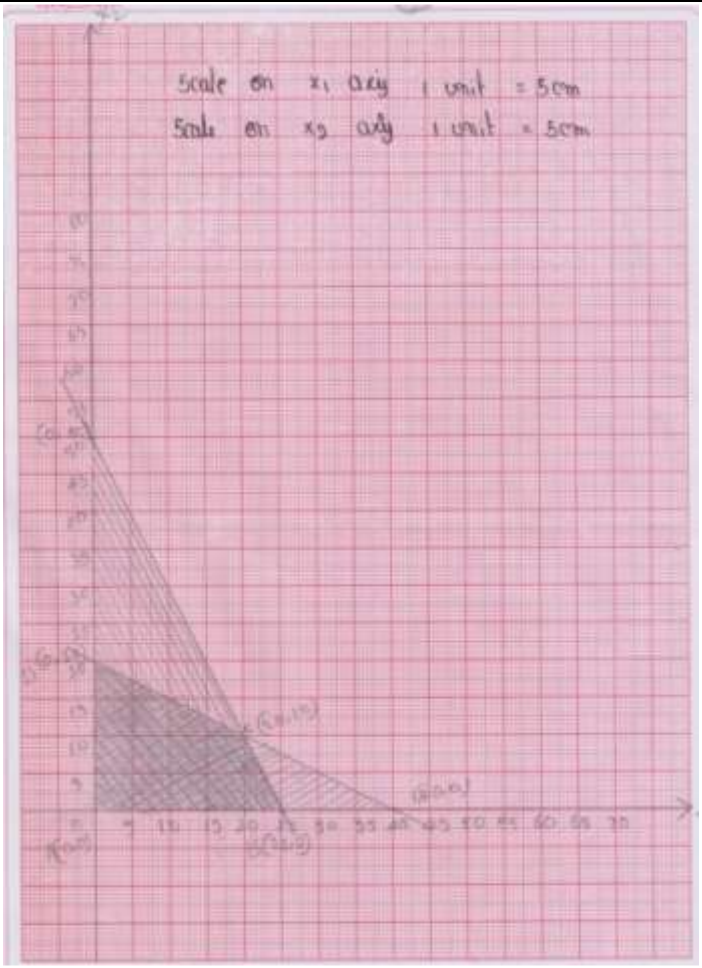
$x + 2y = 40$

x	0	40
y	20	0

$x = 0, 0 + 2y = 40 \Rightarrow y = 20$

$y = 0, x + 2(0) = 40 \Rightarrow x = 40$

$\max Z = x + y$



From graph

$Z(A) = 0 + 0 = 0$

$Z(B) = 25 + 0 = 25$

$Z(C) = 20 + 10 = 30$

$Z(D) = 0 + 20 = 20$

Therefore maximum value of Z occurs at $C = (20,10)$

$Z\ maximum = 30$

7. $Z = 3x + 4y$ Subject to $2x + y \leq 80, 4x + 6y \leq 240$

Sol: Given $2x + y = 80$ …. (1)

x	0	40
y	80	0

When $x = 0$

$0 + y = 80$

$y = 80$

When $y = 0$

$2x + 0 = 80$

$2x = 80$

$x = 40$

$4x + 6y = 240$ …. (2)

When $x = 0$

x	0	60
y	40	0

$4(0) + 6y = 240$

$6y = 240$

$y = 40$

When $y = 0$

$4x + 6(0) = 240$

$4x = 240$

$x = 60$

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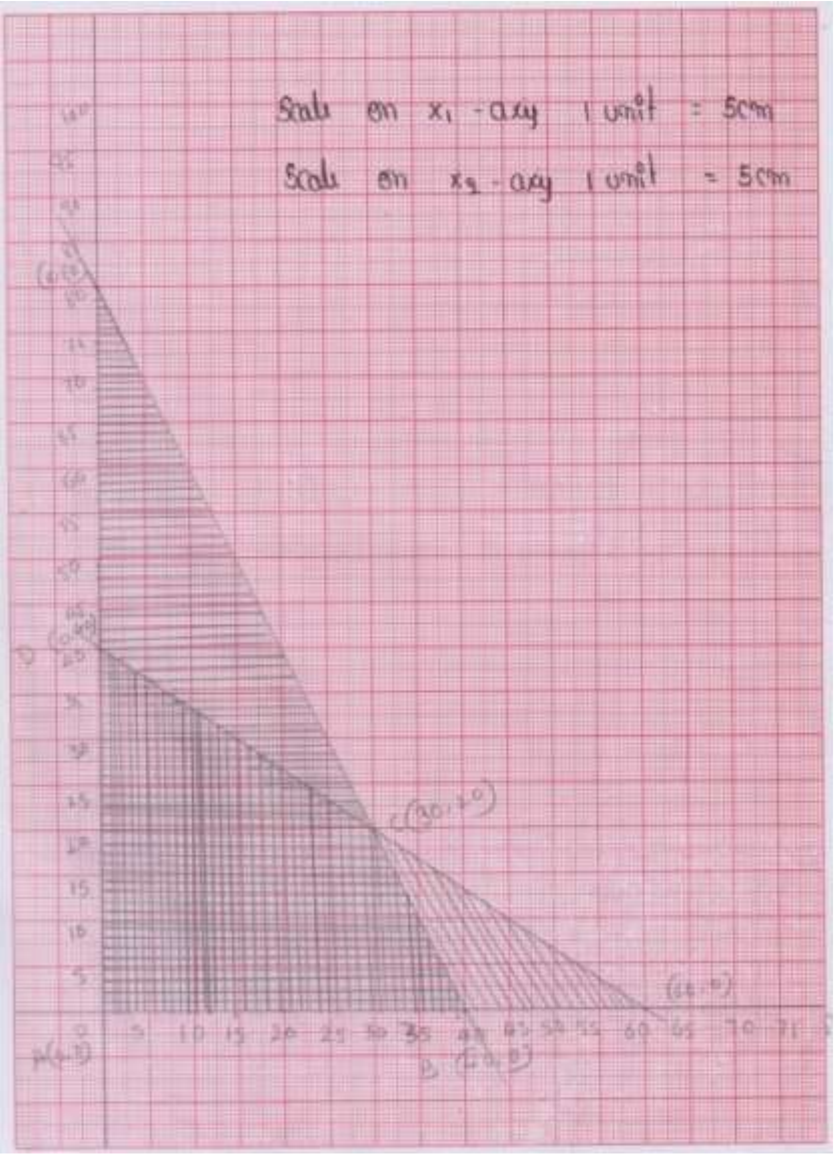


From graph

$Z(A) = 3(0) + 4(0) = 0$

$Z(B) = 3(40) + 4(0) = 120$

$Z(C) = 3(30) + 4(20) = 90 + 80 = 170$

$Z(D) = 3(0) + 4(40) = 0 + 160 = 160$

Therefore maximum value of Z occurs at $C = (30,20)$

$Z\ maximum = 170$

8. $Z = 6x + y$ Subject to $5x + 10y \leq 350, 5x + 4y \leq 200$

$5x + 10y = 350$ …. (1)

x	0	70
y	35	0

When $x = 0$

$0 + 10y = 350$

$y = 35$

When $y = 0$

$5x + 10(0) = 350$

$x = 70$

$5x + 4y = 200$ …. (2)

x	0	40
y	50	0

When $x = 0$

$5(0) + 4y = 200$

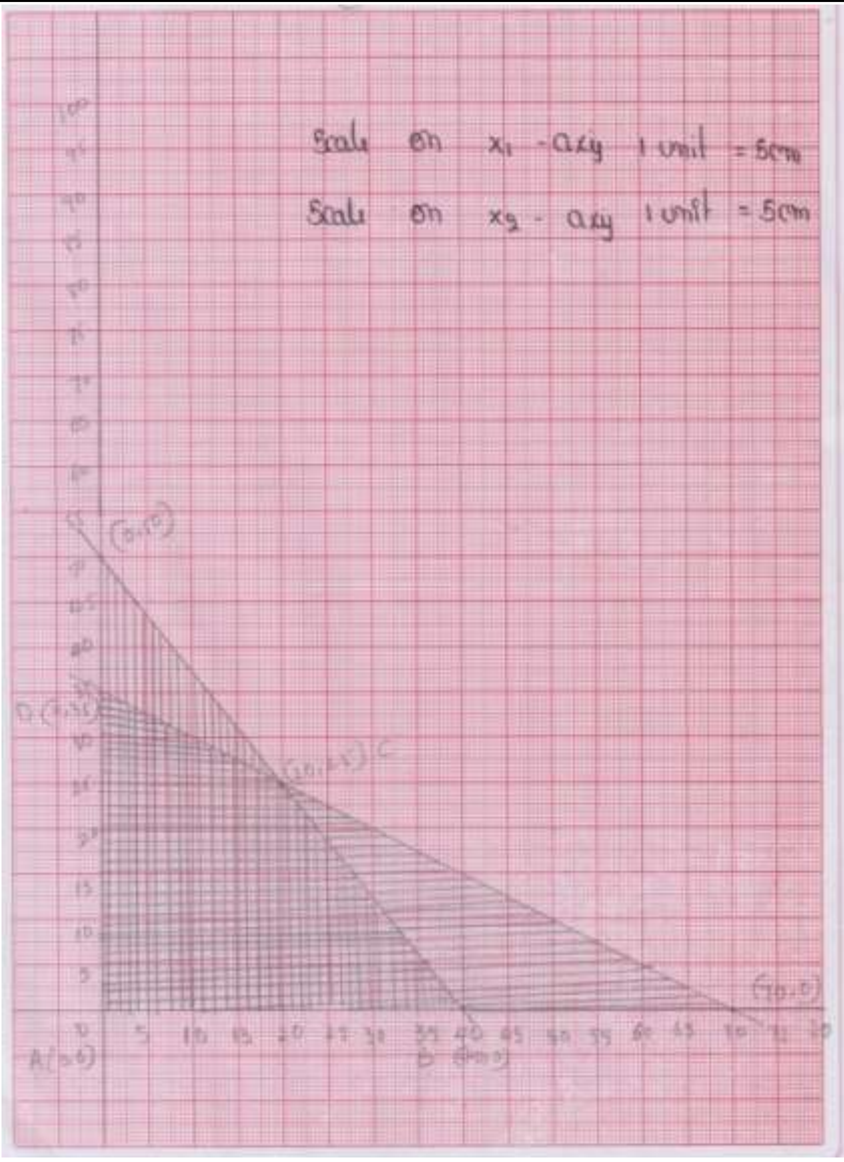
$4y = 200$

$y = 50$

When $y = 0$

$5x + 4(0) = 200$

$5x = 200 \Rightarrow x = 40$



From Graph

$Z = 6x + y$

$Z(A) = 6(0) + 0 = 0$

$Z(B) = 6(40) + 0 = 240$

$Z(C) = 6(20) + 25 = 120 + 25 = 145$

$Z(D) = 6(0) + 50 = 0 + 50 = 50$

Therefore maximum value of Z occurs at $B = (40,0)$

$Z\ maximum = 240$

9. The objective function is parallel to a constraint line

Maximize $Z = 40x_1 + 30x_2$ **.... (*)**

Subject to $1x_1 + 2x_2 \leq 40$ .... (1)

$4x_1 + 3x_2 \leq 120$ ... (2)

$x_1, x_2 > 0$

Where

x_1 = number of bowls

x_2 = number of mugs

Eq (1)

$x_1 = 0, x_2 = ?$

$1x_1 + 2x_2 \leq 40$

$1(0) + 2x_2 = 40$

x_1	0	40
x_2	20	0

$2x_2 = 40 \Rightarrow x_2 = 20$

$x_2 = 0, x_1 = ?$

$1x_1 + 2(0) = 40 \Rightarrow x_2 = 40$

Take equation (2)

$4x_1 + 3x_2 = 12$

If $x_1 = 0$

$4(0) + 3x_2 = 120$

$3x_2 = 120 \Rightarrow x_2 = 40$

x_1	0	30
x_2	40	0

If $x_2 = 0$

$4x_1 + 3(0) = 120$

$\Rightarrow 4x_1 + 0 = 120 \Rightarrow x_1 = 30$

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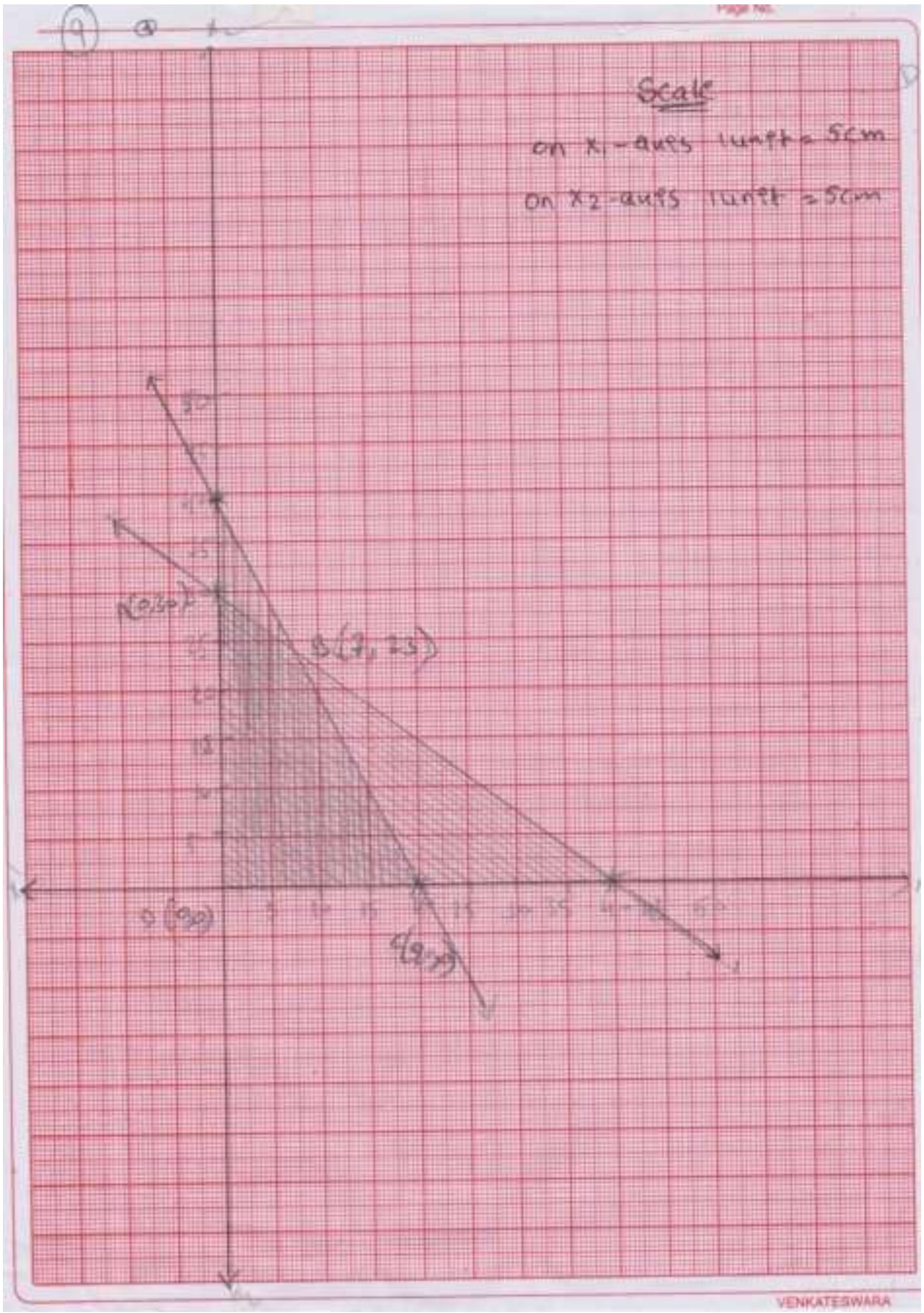
From equation (*)

$Z = 40x_1 + 30x_2$



From Graph

$Z(A) = 40(0) + 30(30) = 900$

$Z(B) = 40(7) + 30(23) = 970$

$Z(C) = 40(20) + 30(0) = 800$

$Z(D) = 40(0) + 30(0) = 0$

Therefore maximum value of Z occurs at B $(7,23)$

Therefore $Z\ maximum = 970$

10. $\boldsymbol{3x_1 + x_2 \leq 7}$

$\boldsymbol{x_1 + 2x_2 \leq 9}$

$\boldsymbol{x_1 \geq 0, x_2 \geq 0}$

Maximum for $\boldsymbol{Z = 2x_1 + x_2}$ **.... (*)**

Sol: Given equation $3x_1 + x_2 \leq 7$ ... (1)

$x_1 + 2x_2 \leq 9$ .... (2)

$x_1 \geq 0, x_2 \geq 0$

From (1)

$3x_1 + x_2 \leq 7$

x_1	0	2.33
x_2	7	0

$3x_1 + x_2 = 7$

$3(0) + x_2 = 7$

$x_2 = 7$

$3x_1 + x_2 = 7$

$3x_1 + 0 = 7$

$x_1 = \frac{7}{3} = 2.33$

From equation (2)

$x_1 + 2x_2 = 9$

x_1	0	9
x_2	4.5	0

$0 + 2x_2 = 9$

$2x_2 = 9$

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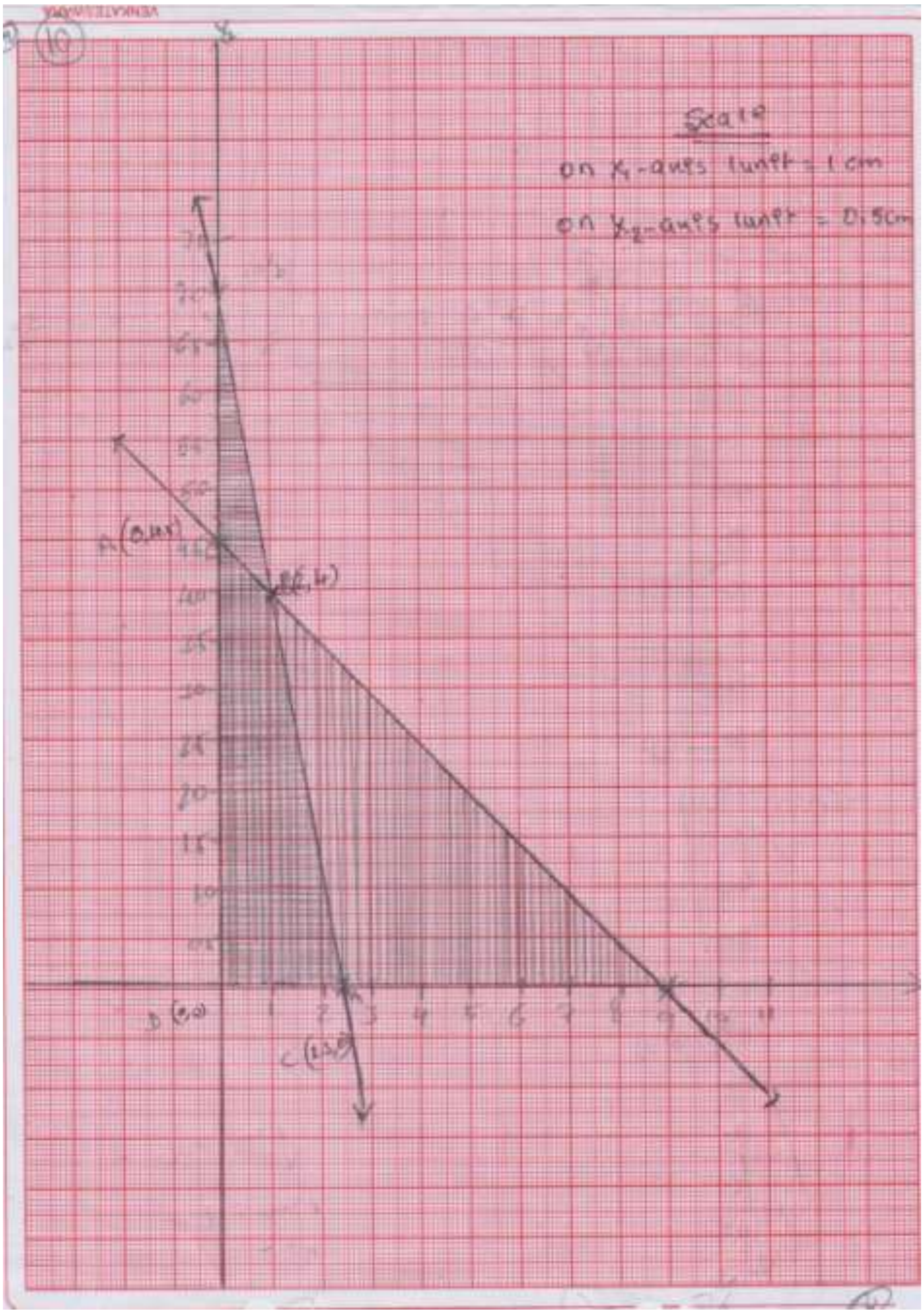
$x_2 = \frac{9}{2} = 4.5$

From equation * $Z = 2x_1 + x_2$



From Graph

$Z(A) = 2(0) + 4.5 = 4.5$

$Z(B) = 2(1) + 4 = 6$

$Z(C) = 2\,(2.33) + 0 = 4.66$

$Z(D) = 2(0) + 0 = 0$

Therefore maximum value of Z is $B(1,4)$

Therefore Z maximum is 6

11. $2x_1 + x_2 \leq 20$

$4x_1 + 5x_2 \leq 60$

$x_1 \geq 0, x_2 \geq 0$

Maximum for $Z = x_1 + 2x_2$

Sol: Given equation $2x_1 + x_2 \leq 20$ .... (1)

$4x_1 + 5x_2 \leq 10$ .... (2)

Maximum for $Z = x_1 + 2x_2$ ... (*)

From (1)

$2x_1 + x_2 = 20$

x_1	0	10
x_2	20	0

$2(0) + x_2 = 20$

$x_2 = 20$

$2x_1 + x_2 = 20$

$2x_1 + 0 = 20 \Rightarrow 2x_1 = 20 \Rightarrow x_1 = \frac{20}{2} = 10$

From (2)

$4x_1 + 5x_2 = 60$

x_1	0	15
x_2	12	0

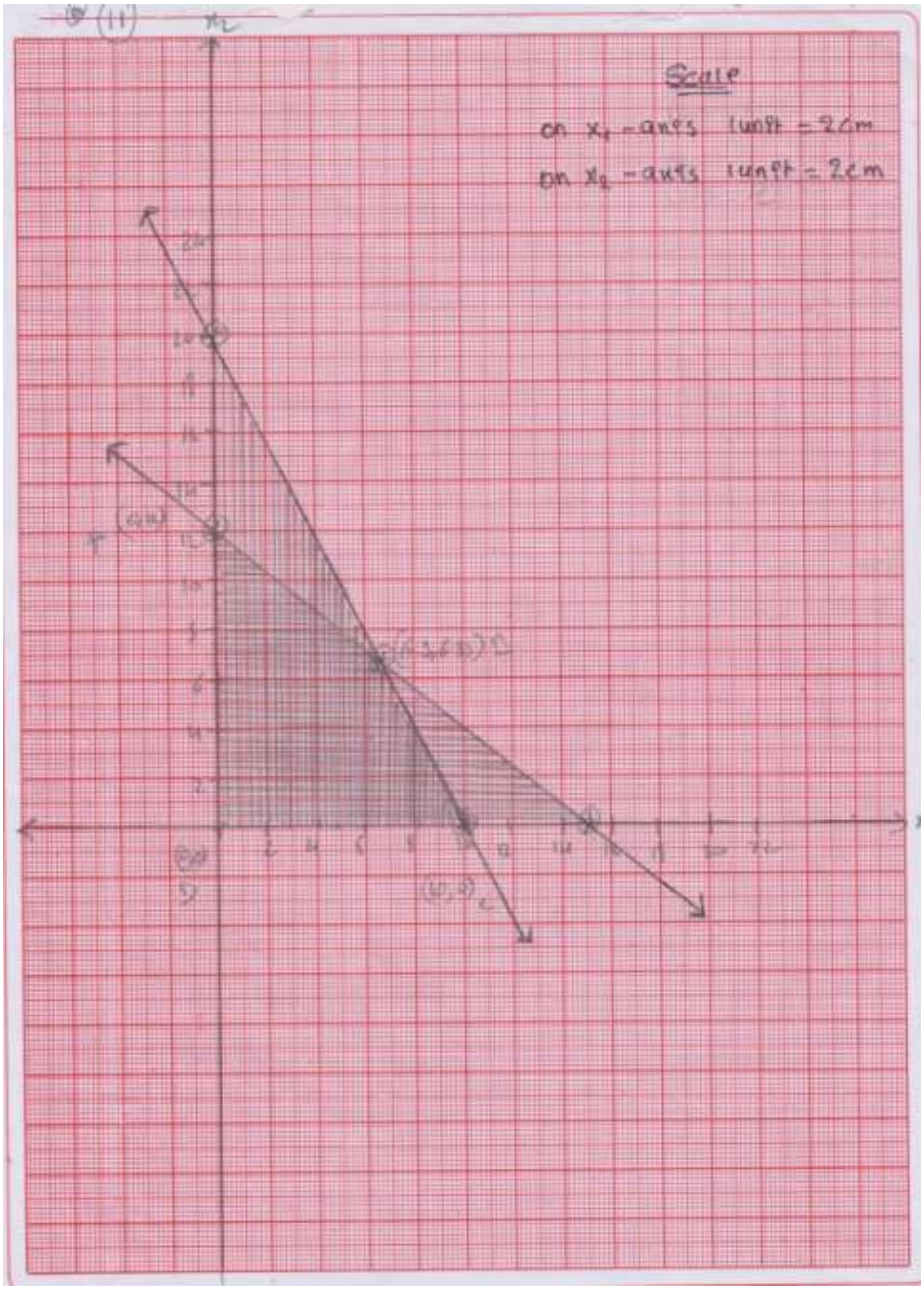
$4(0) + 5x_2 = 60$

$5x_2 = 60 \Rightarrow x_2 = \frac{60}{5} = 12$

$4x_1 + 5x_2 = 60$

$4x_1 + 5(0) = 60 \Rightarrow 4x_1 = 60 \Rightarrow x_1 = \frac{60}{4} = 15$

From *, $Z = x_1 + 2x_2$



From Graph

$Z(A) = 0 + 2(12) = 24$

$Z(B) = 6.3 + 2(6.3) = 18.9$

$Z(C) = 10 + 2(0) = 10$

$Z(D) = 0 + 2(0) = 0$

Therefore Z is occur in point $A(0,12)$

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Therefore Z maximum is 24

Solving the following LPP using graphical method.

12. Minimization $Z = 20x_1 + 10x_2$ subjection to

$x_1 + 2x_2 \leq 40$

$3x_1 + x_2 \geq 30$

$4x_1 + 3x_2 \geq 60$

Sol: Given equation $x_1 + 2x_2 = 40$ …. (1)

When $x_1 = 0 \Rightarrow x_2 = 20$

x_1	0	40
x_2	20	0

When $x_2 = 0 \Rightarrow x_1 = 40$

$3x_1 + x_2 = 30$ …. (2)

When $x_1 = 0 \Rightarrow x_2 = 30$

x_1	0	10
x_2	30	0

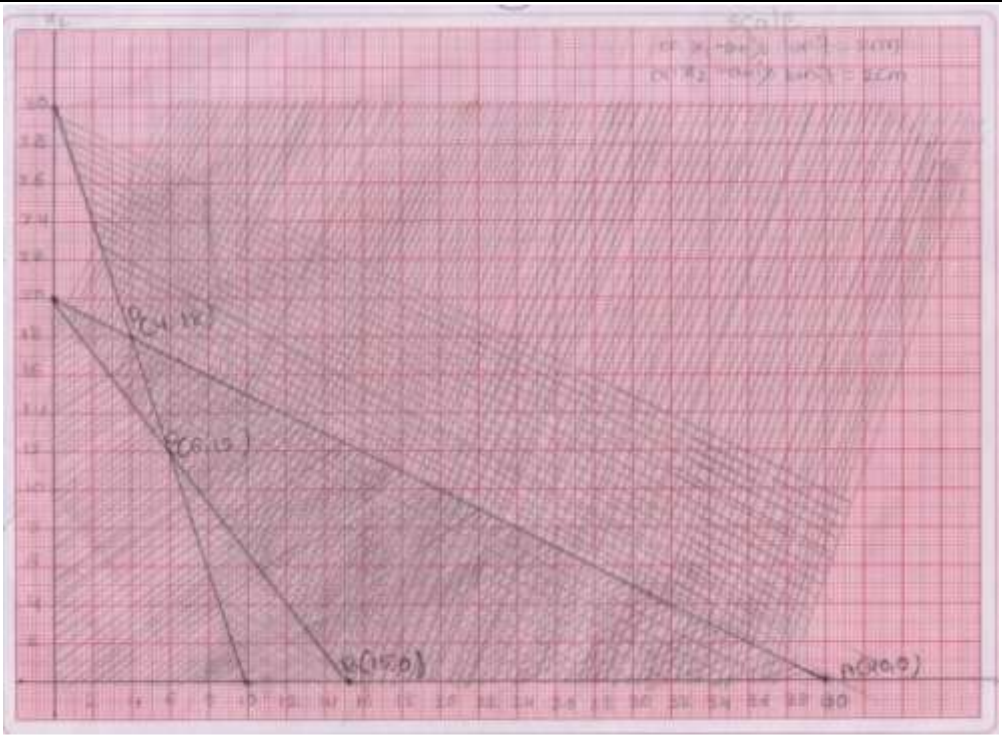
When $x_2 = 0 \Rightarrow x_1 = 10$

$4x_1 + 3x_2 = 60$ …. (3)

When $x_1 = 0 \Rightarrow x_2 = 20$

x_1	0	15
x_2	20	0

When $x_2 = 0 \Rightarrow x_1 = 15$



From Graph: Minimization $Z = 20x_1 + 10x_2$

$A(40{,}0) \Rightarrow Z = 20(40) + 10(0) = 800$

$B(15{,}0) \Rightarrow Z = 20(15) + 10(0) = 300$

$C(6{,}12) \Rightarrow Z = 20(16) + 10(12) = 240$

$D(4{,}18) \Rightarrow Z = 20(4) + 10(18) = 260$

$Z(Minimization) = 240$

Optical Solution $C(6{,}12)$

13. Solving the following LPP using graphical method.

Minimization $Z = 4x_1 + 6x_2$ subjection to

$\boldsymbol{x_1 + x_2 \geq 8}$

$\boldsymbol{6x_1 + x_2 \geq 12}$

$\boldsymbol{x_1 and\ x_2 \geq 0}$

Sol: $x_1 + x_2 = 8$ …. (1)

When $x_1 = 0 \Rightarrow x_2 = 8$

x_1	0	8
x_2	8	0

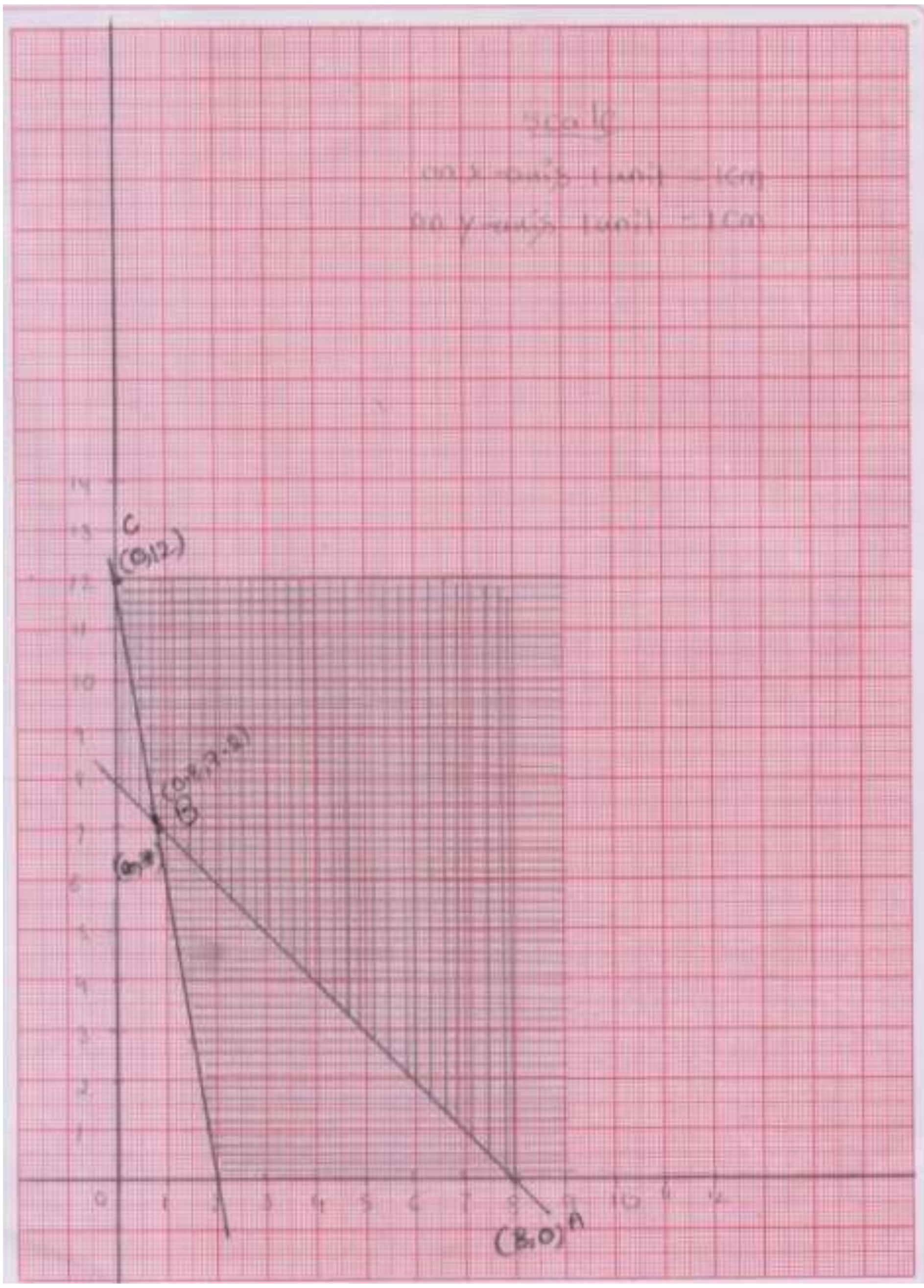
When $x_2 = 0 \Rightarrow x_1 = 8$

$6x_1 + x_2 = 12$ .... (2)

When $x_1 = 0 \Rightarrow x_2 = 12$

x_1	0	2
x_2	12	0

When $x_2 = 0 \Rightarrow x_1 = 2$



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From Graph: Minimization $Z = 4x_1 + 6x_2$

$A(8,0) = 4(8) + 6(0) = 32$

$B(0.8,7.2) = 4(0.8) + 6(7.2) = 46.4$

$C(0,12) = 4(0) + 6(12) = 72$

Optical Solution $A(8,0)$

$Z(optimum) = 32$

14. Solving the following LPP using graphical method.

Minimization $Z = 3x_1 + 2x_2$ subjection to

$\mathbf{5x_1 + x_2 \geq 10}$

$\mathbf{x_1 + x_2 \geq 6}$

$\mathbf{x_1 + 4x_2 \geq 12}$

Sol: $5x_1 + x_2 = 10$ …. (1)

When $x_1 = 0 \Rightarrow x_2 = 10$

x_1	0	2
x_2	10	0

When $x_2 = 0 \Rightarrow x_1 = 2$

$x_1 + x_2 = 6$ …. (2)

When $x_1 = 0 \Rightarrow x_2 = 6$

x_1	0	6
x_2	6	0

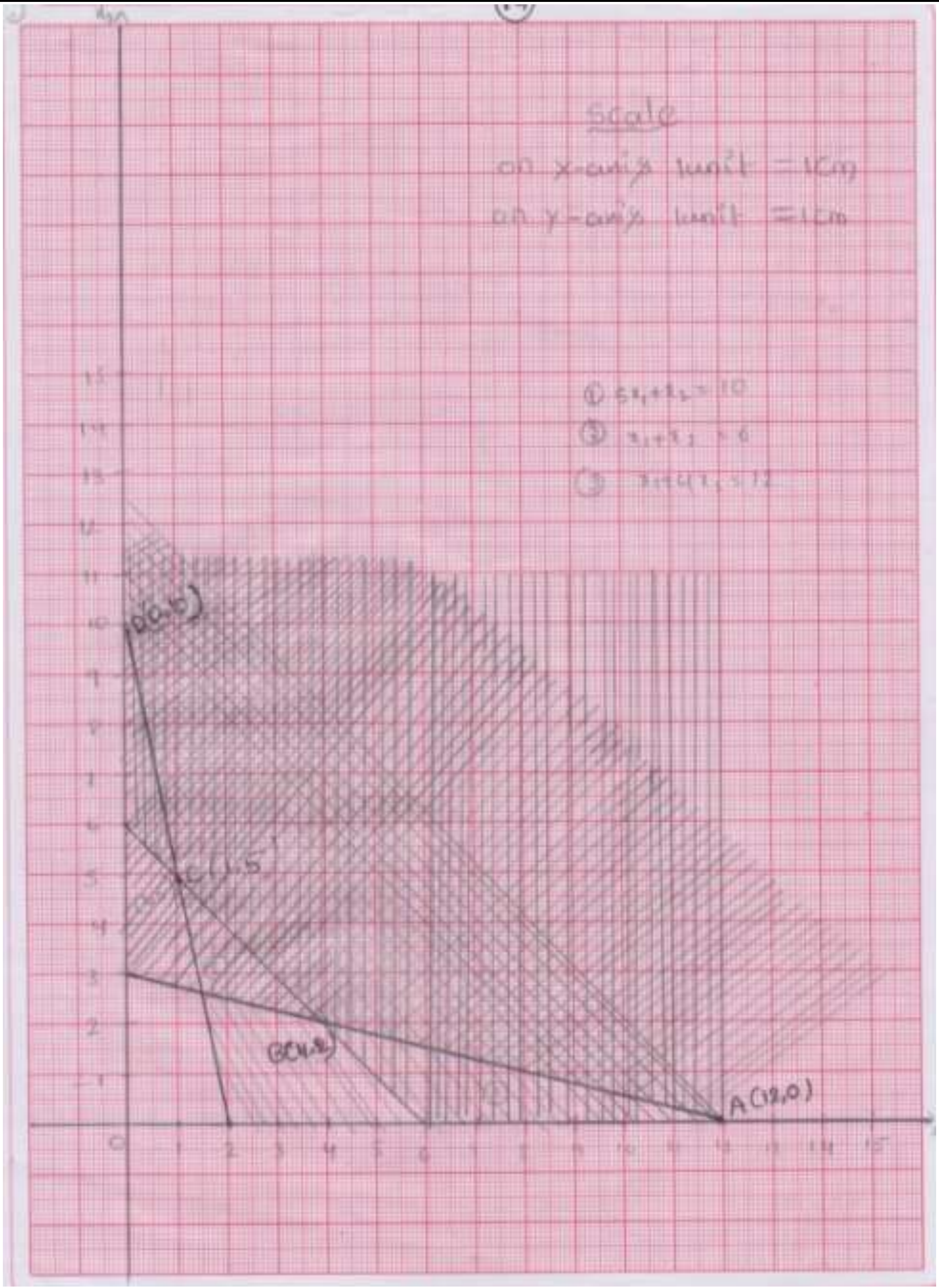
When $x_2 = 0 \Rightarrow x_1 = 6$

$x_1 + 4x_2 = 12$ …. (3)

When $x_1 = 0 \Rightarrow x_2 = 3$

x_1	0	12
x_2	3	0

When $x_2 = 0 \Rightarrow x_1 = 12$



From Graph: Minimization $Z = 3x_1 + 2x_2$

$A(12,0) \Rightarrow 3(12) + 2(0) = 36$

$B(4,2) \Rightarrow 3(4) + 2(2) = 16$

$C(1,5) \Rightarrow 3(1) + 2(5) = 13$

$D(0,10) \Rightarrow 3(0) + 2(10) = 20$

Optimum solution $C(1,5)$

$Z(optimum) = 13$

15. Solving the following LPP using graphical method.

Minimization $Z = 18x + 10y$ subjection to

$\boldsymbol{4x + y \geq 20}$

$\boldsymbol{2x + 3y \geq 30}$

$\boldsymbol{x, y \geq 0}$

Sol: $4x + y = 20$ …. (1)

When $x = 0 \Rightarrow y = 20$

x	0	5
y	20	0

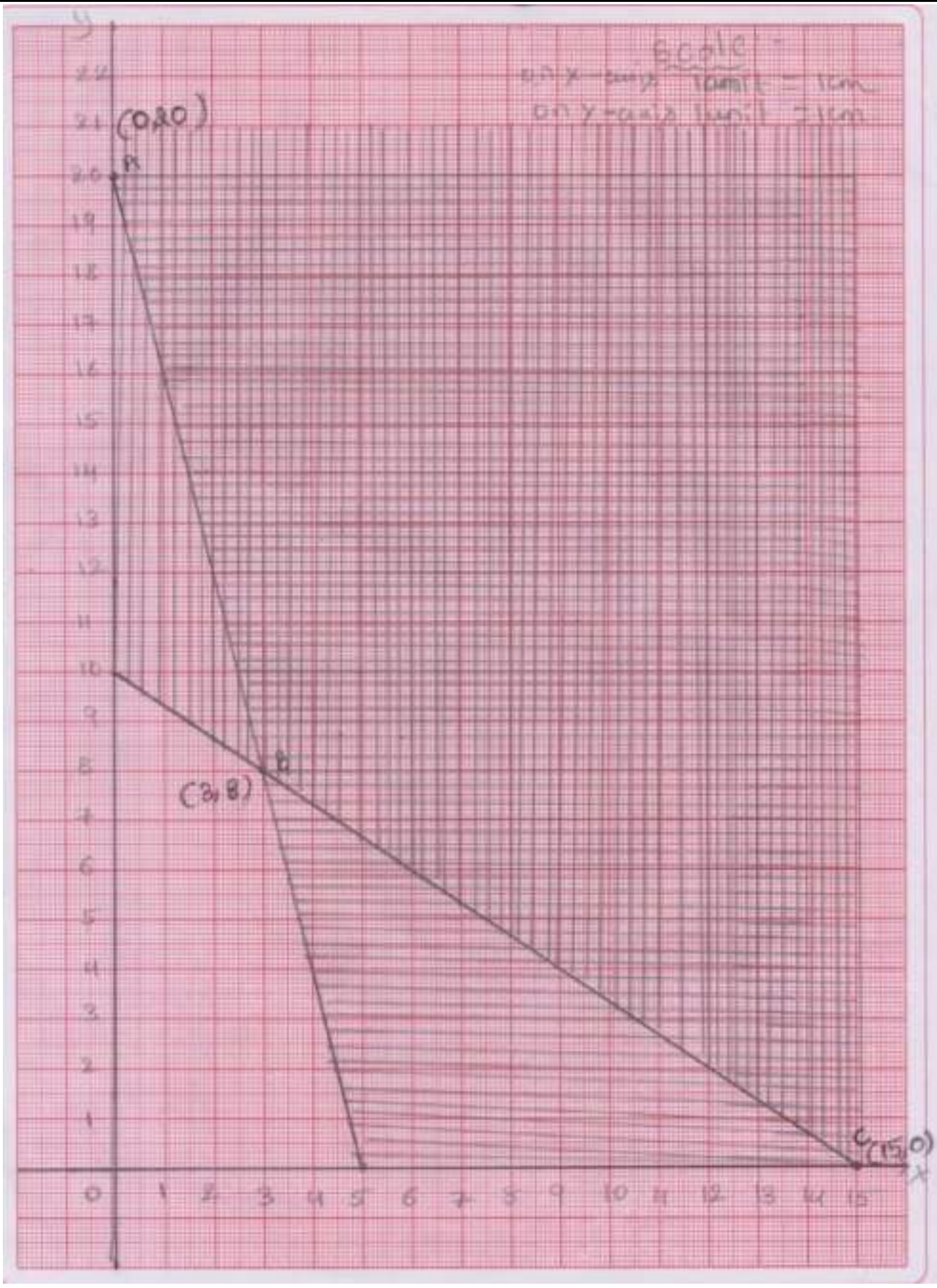
When $y = 0 \Rightarrow x = 5$

$2x + 3y = 30$ …. (2)

When $x = 0 \Rightarrow y = 10$

x	0	15
y	10	0

When $y = 0 \Rightarrow x = 15$



From Graph:

Minimization $Z = 18x + 10y$

$A(0,20) \Rightarrow Z = 0 + 10(20) = 200$

$B(3,8) \Rightarrow Z = 18(3) + 10(8) = 134$

$C(15,0) \Rightarrow Z = 18(15) + 10(0) = 270$

Optical Solution $B(3,8)$

$Z\ (optimum) = 134$

16. Solving the following LPP using graphical method.

Minimization $Z = 7x + 5y$ subjection to

$4x + 3y \geq 240$

$2x + y \geq 100$

$x, y \geq 0$

Sol: $4x + 3y = 240$ …. (1)

When $x = 0 \Rightarrow y = 80$

x	0	60
y	80	0

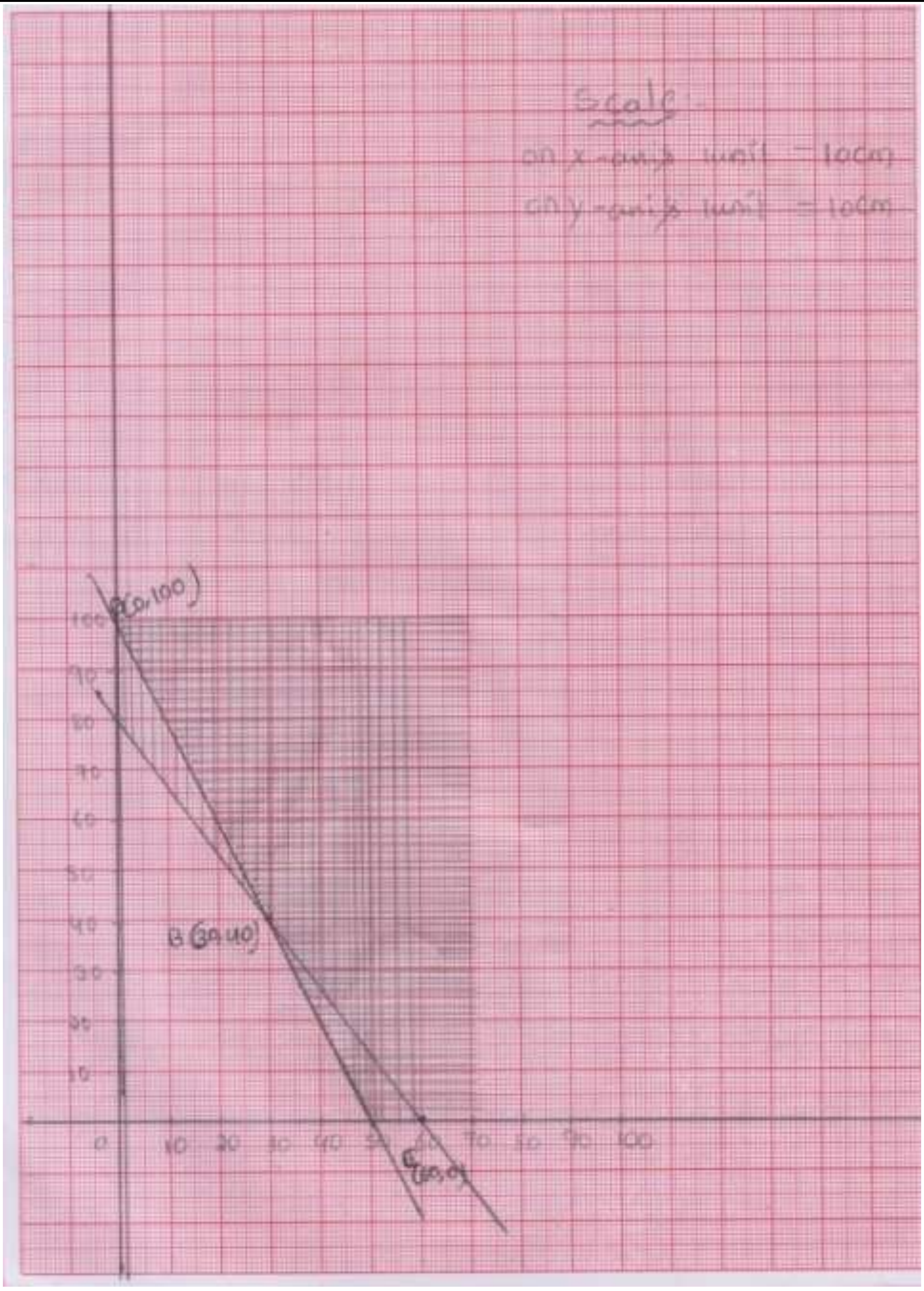
When $y = 0 \Rightarrow x = 60$

$2x + y = 100$ …. (2)

When $x = 0 \Rightarrow y = 100$

x	0	50
y	100	0

When $y = 0 \Rightarrow x = 50$



From Graph:

Minimization $Z = 7x + 5y$

$A(0{,}100) \Rightarrow Z = 7(0) + 5(100) = 500$

$B(30{,}40) \Rightarrow Z = 7(30) + 5(40) = 410$

$C(60{,}0) \Rightarrow Z = 7(60) + 5(0) = 420$

Optical Solution $B(30{,}40)$

$Z\ (optimum) = 410$

17. Solving the following LPP using graphical method.

Minimization $Z = 5x_1 + 4x_2$ subjection to

$\mathbf{4x_1 + x_2 \geq 40}$

$\mathbf{2x_1 + 3x_2 \geq 90}$

$\mathbf{x_1, x_2 \geq 0}$

Sol: $4x_1 + 3x_2 = 40$ .... (1)

When $x_1 = 0 \Rightarrow x_2 = 40$

x_1	0	10
x_2	40	0

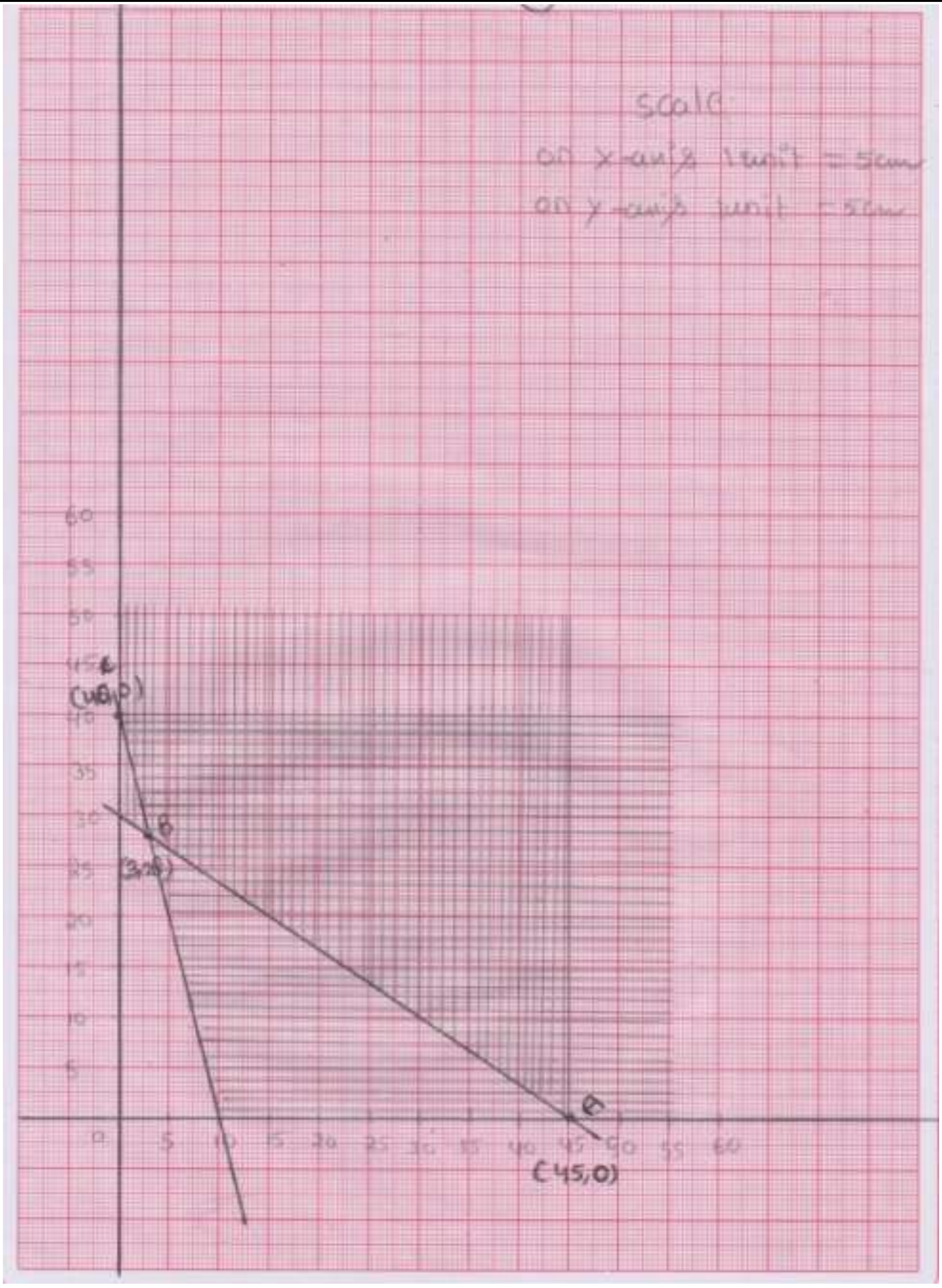
When $x_2 = 0 \Rightarrow x_1 = 10$

$2x_1 + 3x_2 = 90$ .... (2)

When $x_1 = 0 \Rightarrow x_2 = 30$

x_1	0	45
x_2	30	0

When $x_2 = 0 \Rightarrow x_1 = 45$



From Graph:

Minimization $Z = 5x_1 + 4x_2$

$A(45{,}0) \Rightarrow 5(45) + 4(0) = 225$

$B(3{,}28) \Rightarrow 5(3) + 4(28) = 127$

$C(0{,}40) \Rightarrow 5(0) + 4(40) = 160$

Optical Solution $B(3{,}28)$

$Z\ (optimum) = 127$

18. Solving the following LPP using graphical method.

Minimization $Z = 20x_1 + 40x_2$ subjection to

$36x_1 + 6x_2 \geq 108$

$3x_1 + 12x_2 \geq 36$

$20x_1 + 10x_2 \geq 100$

$x_1, x_2 \geq 0$

Sol: $36x_1 + 6x_2 = 108$ …. (1)

When $x_1 = 0 \Rightarrow x_2 = 18$

x_1	0	3
x_2	18	0

When $x_2 = 0 \Rightarrow x_1 = 3$

$3x_1 + 12x_2 = 36$ …. (2)

When $x_1 = 0 \Rightarrow x_2 = 3$

x_1	0	12
x_2	3	0

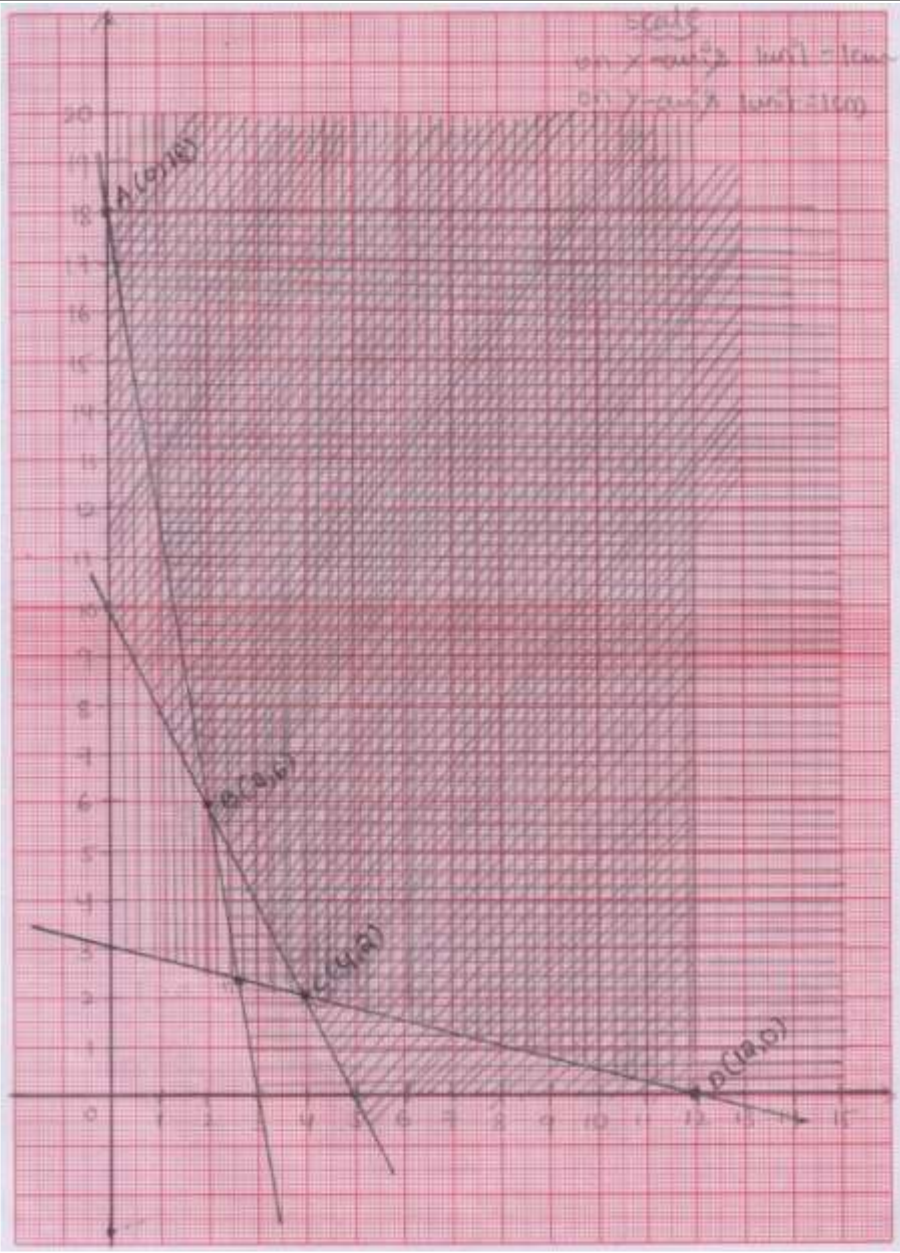
When $x_2 = 0 \Rightarrow x_1 = 12$

$20x_1 + 10x_2 = 100$ …. (3)

When $x_1 = 0 \Rightarrow x_2 = 10$

x_1	0	5
x_2	10	0

When $x_2 = 0 \Rightarrow x_1 = 5$



From Graph:

Minimization $Z = 20x_1 + 40x_2$

$A(0,18) \Rightarrow 20(0) + 40(18) = 720$

$B(2,6) \Rightarrow 20(2) + 40(6) = 280$

$C(4,2) \Rightarrow 20(4) + 40(2) = 160$

$D(12,0) \Rightarrow 20(12) + 40(0) = 240$

Optical Solution $C(4,2)$

$Z\ (optimum) = 160$

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19. Solving the following LPP using graphical method.

Minimization $Z = 4x_1 + 3x_2$ subjection to

$2x_1 + x_2 = 40$

$x_1 + 2x_2 = 50$

$x_1 + x_2 = 35$

Sol: $2x_1 + x_2 = 40$ …. (1)

When $x_1 = 0 \Rightarrow x_2 = 40$

x_1	0	20
x_2	40	0

When $x_2 = 0 \Rightarrow x_1 = 20$

$x_1 + 2x_2 = 50$ …. (2)

When $x_1 = 0 \Rightarrow x_2 = 25$

x_1	0	50
x_2	25	0

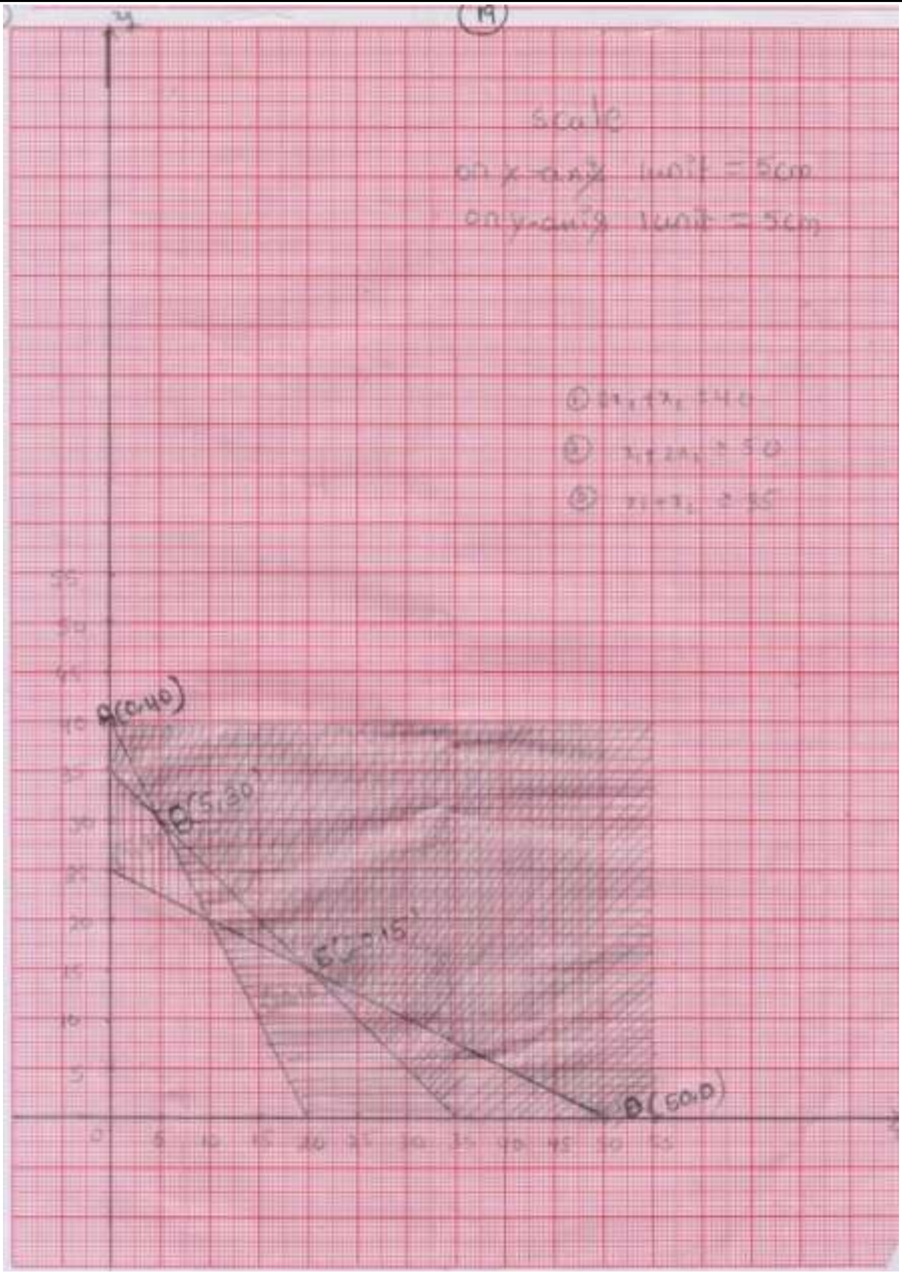
When $x_2 = 0 \Rightarrow x_1 = 50$

$x_1 + x_2 = 35$ …. (3)

When $x_1 = 0 \Rightarrow x_2 = 35$

x_1	0	35
x_2	35	0

When $x_2 = 0 \Rightarrow x_1 = 35$



From Graph:

Minimization $Z = 4x_1 + 3x_2$

$A(0,40) \Rightarrow Z = 4(0) + 3(40) = 120$

$B(5,30) \Rightarrow Z = 4(5) + 3(30) = 20 + 90 = 110$

$C(20,15) \Rightarrow Z = 4(20) + 3(15) = 80 + 45 = 125$

$D(50,0) \Rightarrow Z = 4(50) + 3(0) = 200$

Optical Solution $B(5,30)$

$Z\ (optimum) = 110$

Conclusion

Linear Programming is a powerful mathematical tool for optimizing the allocation of limited resources under a set of linear constraints. This paper has presented the theoretical foundations of Linear Programming together with a systematic graphical solution methodology for solving two-variable optimization problems. Through numerous maximization and minimization examples, the study demonstrates how feasible regions are constructed and how optimal solutions are obtained by evaluating the extreme points of the feasible region. The graphical method not only simplifies the solution process but also provides a clear visual interpretation of feasible, infeasible, bounded, and unbounded solution spaces. Although its application is limited to problems involving two decision variables, the graphical approach serves as an excellent educational technique for understanding the concepts underlying more advanced optimization methods such as the Simplex algorithm. The concepts and examples presented in this paper highlight the wide applicability of Linear Programming in engineering, economics, business management, transportation, manufacturing, and operations research. Future work may extend these techniques to higher-dimensional optimization problems using computational methods and optimization software, enabling the solution of large-scale real-world decision-making problems.

References

1. **Bazaraa, M. S., Jarvis, J. J., & Sherali, H. D. (2010). Linear Programming and Network Flows (4th ed.). John Wiley & Sons.**
2. **Dantzig, G. B. (1963). Linear Programming and Extensions. Princeton University Press.**
3. **Hamdy, A. T. (2017). Operations Research: An Introduction (10th ed.). Pearson Education.**
4. **Hillier, F. S., & Lieberman, G. J. (2021). Introduction to Operations Research (11th ed.). McGraw-Hill Education.**
5. **Kanti Swarup, Gupta, P. K., & Man Mohan. (2019). Operations Research (18th ed.). Sultan Chand & Sons.**
6. **Taha, H. A. (2017). Operations Research: An Introduction (10th ed.). Pearson.**
7. **Winston, W. L. (2004). Operations Research: Applications and Algorithms (4th ed.). Thomson Brooks/Cole.**
8. **Chvátal, V. (1983). Linear Programming. W. H. Freeman and Company.**
9. **Vanderbei, R. J. (2020). Linear Programming: Foundations and Extensions (5th ed.). Springer.**
10. **Murty, K. G. (1983). Linear Programming. John Wiley & Sons.**
11. **Schrijver, A. (1998). Theory of Linear and Integer Programming. John Wiley & Sons.**
12. **Bertsimas, D., & Tsitsiklis, J. N. (1997). *Introduction to Linear Optimization*. Athena Scientific.**